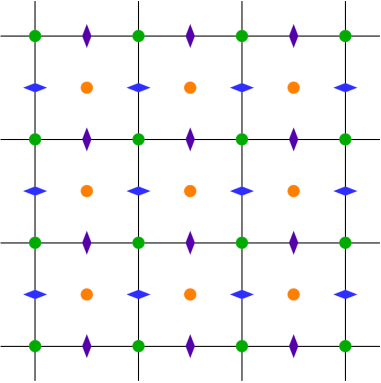
Discrete Operators and Implementation

These operators act on data located in different spaces, as indicated on the control volume document. Let the following notation signify that the operator operates on data that live on and stores the result on .

# Order of accuracy

All of the below operators should be of 2nd order. The difference formulae do not look like a collocated 2nd order difference formula because of the staggered grid. This section shows how these seemingly 1st order accurate differences are in fact 2nd order.



# 1st derivative of data living on , for

We may approximate the pressure at the cell center location about the face . We may write this as a Taylor expansion as

Similarly, we may approximate

Subtracting these results together we have

Or

Assuming the face is half-way between the cell centers, we have which yields

Note that lives on the face corresponding to , and the two pressures on the RHS correspond to the finite difference formula

The second order accurate first derivative.

# 1st derivative of data living on F, for

We may approximate the velocity at the cell face location about the node and . We may write this as a Taylor expansion as

Similarly, we may approximate

Subtracting these results together we have

Or

Assuming the face is half-way between the cell centers, we have which yields

The second order accurate first derivative. Note that lives on the cell center.

# 2nd derivative of data living on , for

The 2nd derivative of data living on is the same as an ordinary collocated grid, however, the boundaries must be modified accordingly. See the last section for details.

# 2nd derivative of data living on , for

The 2nd derivative of data living on is the same as an ordinary collocated grid.

# Operators and Spaces

## Divergence

Where and . This is the analog of .

## Cell centered Gradient

Where and . This is the analog of .

## edgeCurl (nodeCurl in 2D)

Where and . This is the discrete analog of .

## faceCurl (edgeCurl in 2D)

Where and . This is the discrete analog of .

## edgeLaplacian (nodeLaplacian in 2D)

Where and . This is the discrete analog of .

## Cell-centered Laplacian

Where and . This is the discrete analog of .

# Averages

## edgeAverage

## faceAverage

# Implementation details of boundary derivatives

The implementation details have been written in terms on normal and tangential components of the operator and vector field quantities. For example,

These two derivatives must be calculated using a modified stencil near the boundary since the data that live tangent wrt the boundary do not intersect with the boundary. These modified stencils can be computed by using Taylor expansions on and near the boundary.

## Modified boundary stencils for first order derivatives of face-based data

## Normal derivatives

### Interior (CD2)

### Start Boundary (Ordinary upwind)

### End Boundary (Ordinary downwind)

## Tangential derivatives

### Interior (CD2)

### Start Boundary (modified stencil)

### End Boundary (modified stencil)

## Modified boundary stencils for second order derivatives of face-based data

## Normal derivatives

### Interior (CD2)

### Start Boundary (Ordinary upwind)

### End Boundary (Ordinary downwind)

## Tangential derivatives

### Interior (CD2)

### Start Boundary (modified stencil)

### End Boundary (modified stencil)